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SOMBOR INDICES OF CERTAIN GRAPH OPERATORS

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ABSTRACT

Recently, Gutman considered a class of novel graph invariants of which the Sombor index was defined. In this paper, we study the certain Sombor indices and their exponentials of regular and complete bipartite graphs using some graph operators.

Keywords: *Sombor index, Sombor exponential, reduced Sombor index, reduced Sombor exponential, line graph, subdivision graph.*
Mathematics Subject Classification: *05C05, 05C07, 05C90.*

I. INTRODUCTION

Let G be a finite, simple, connected graph with vertex set $V(G)$ and edge set $E(G)$. The degree of a vertex u is denoted by $d_G(u)$. The edge connecting the vertices u and v will be denoted by uv. We refer [1] for undefined notations and terminologies.

The Sombor index of a graph G was introduced by Gutman in [2] and defined it as

$$SO(G) = \sum_{uv\in E(G)} \sqrt{\sigma_n(u)^2 + \sigma_n(v)^2}.$$

In [3], Kulli introduced the first (a, b)-KA index of a graph G, defined it as

$$K^1_{a,b}(G) = \sum_{uv\in E(G)} \left[d_G(u)^a + d_G(v)^a\right].$$

Clearly, the Sombor index is obtained as special case of the first (a, b)-KA index. If a=2 and b=½, then $K^1_{2,\frac{1}{2}}(G) = SO(G)$.

Considering the Sombor index, we introduce the Sombor exponential of a graph G, defined as

$$SO(G,x) = \sum_{uv\in E(G)} x^{\sqrt{d_G(u)^2 + d_G(v)^2}}.$$

In [2], Gutman defined the reduced Sombor index of a graph G as

$$RSO(G) = \sum_{uv\in E(G)} \sqrt{\left(d_G(u)-1\right)^2 + \left(d_G(v)-1\right)^2}.$$

Considering the reduced Sombor index, we propose the reduced Sombor exponential of a graph G and it is defined as

$$RSO(G,x) = \sum_{uv\in E(G)} x^{\sqrt{\left(d_G(u)-1\right)^2 + \left(d_G(v)-1\right)^2}}.$$

The average Sombor index was proposed by Gutman in [2] and it is defined as

$$ASO(G)=\sum_{uv\in E(G)}\sqrt{\left(d_G(u)-\frac{2m}{n}\right)^2+\left(d_G(v)-\frac{2m}{n}\right)^2}.$$

where $|V(G)| = n$ and $|E(G)| = m$.

This index is equal to zero for regular graphs.

Considering the average Sombor index, we introduce the average Sombor exponential of a graph G, defined as

$$ASO(G,x)=\sum_{uv\in E(G)}x^{\sqrt{\left(d_G(u)-\frac{2m}{n}\right)^2+\left(d_G(v)-\frac{2m}{n}\right)^2}}.$$

In Chemical Graph Theory, many graph indices were introduced and studied, see [4, 5]. The reduced first [6] and second [6] Zagreb indices were introduced and studied.

In this paper, we establish some results on the Sombor indices and their corresponding polynomials for line and subdivision graphs of some standard graphs.

2. RESULTS FOR LINE GRAPHS

The line graph $L(G)$ of a graph G is the graph whose vertex set corresponds to the edges of G such that two vertices of $L(G)$ are adjacent if and only if the corresponding edges of G are adjacent.

In the following theorem, we compute the Sombor indices and their exponentials of the line graphs of r-regular graphs.

Theorem 1. Let G be an r-regular graph with $n \geq 2$ vertices. Then

(i) $SO(L(G))=\sqrt{2}nr(r-1)^2$.

(ii) $RSO(L(G))=\frac{1}{\sqrt{2}}nr(r-1)(2r-3)$.

(iii) $ASO(L(G)) = 0$.

(iv) $SO(L(G),x)=\frac{1}{2}nr(r-1)x^{2\sqrt{2}(r-1)}$.

(v) $RSO(L(G),x)=\frac{1}{2}nr(r-1)x^{\sqrt{2}(2r-3)}$.

(vi) $ASO(L(G),x)=\frac{1}{2}nr(r-1)x^{0}$.

Proof: Let G be an r-regular graph with $n\geq2$ vertices. Then the line graph $L(G)$ of G is also an $(2r-2)$-regular graph with $\frac{1}{2}nr$ vertices and $\frac{1}{2}nr(r-1)$ edges. Thus $d_{L(G)}(u) = 2r - 2$ for any vertex u of $L(G)$.

By using definitions, we deduce

(i) $SO(L(G)) = \frac{1}{2}nr(r-1)\sqrt{(2r-2)^2+(2r-2)^2}$
$= \sqrt{2}nr(r-1)^2$.

(ii) $RSO(L(G)) = \frac{1}{2}nr(r-1)\sqrt{(2r-2-1)^2+(2r-2-1)^2}$
$= \frac{1}{\sqrt{2}}nr(r-1)(2r-3)$.

(iii) $ASO(L(G)) = 0$, since L(G) is regular.

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(iv) $SO(L(G), x) = \frac{1}{2}nr(r-1)x^{\sqrt{(2r-2)^2+(2r-2)^2}}$

$= \frac{1}{2}nr(r-1)x^{2\sqrt{2}(r-1)}$.

(v) $RSO(L(G), x) = \frac{1}{2}nr(r-1)x^{\sqrt{(2r-2-1)^2+(2r-2-1)^2}}$

$= \frac{1}{2}nr(r-1)x^{\sqrt{2}(2r-3)}$.

(vi) $ASO(L(G), x) = \frac{1}{2}nr(r-1)x^0$.

From Theorem 1, we obtain the following results.

Corollary 1.1. Let C_n be a cycle with $n\geq 3$ vertices. Then

(i) $SO(L(C_n)) = 2\sqrt{2}n$

(ii) $RSO(L(C_n)) = \sqrt{2}n$

(iii) $ASO(L(C_n)) = 0$.

(iv) $SO(L(C_n), x) = nx^{2\sqrt{2}}$

(v) $RSO(L(C_n), x) = nx^{\sqrt{2}}$

(vi) $ASO(L(C_n), x) = nx^0$

Corollary 1.2. Let K_n be a complete graph with n vertices. Then

(i) $SO(L(K_n)) = \sqrt{2}n(n-1)(n-2)^2$.

(ii) $RSO(L(K_n)) = \frac{1}{\sqrt{2}}n(n-1)(n-2)(2n-5)$

(iii) $ASO(L(K_n)) = 0$.

(iv) $SO(L(K_n), x) = \frac{1}{2}n(n-1)(n-2)x^{2\sqrt{2}(n-2)}$

(v) $RSO(L(K_n), x) = \frac{1}{2}n(n-1)(n-2)x^{\sqrt{2}(2n-5)}$

(vi) $ASO(L(K_n), x) = \frac{1}{2}n(n-1)(n-2)x^0$.

In the following theorem, we determine the Sombor indices and their exponentials of the line graphs of complete bipartite graphs.

Theorem 2. Let $K_{p,q}$ be a complete bipartite graph with $p+q$ vertices, pq edges and $1 \leq p \leq q$. Then

(i) $SO\left(L\left(K_{p,q}\right)\right) = \frac{1}{\sqrt{2}}pq(p+q-2)^2$.

(ii) $RSO\left(L\left(K_{p,q}\right)\right) = \frac{1}{\sqrt{2}}pq(p+q-2)(p+q-3)$.

(iii) $ASO(L(K_{p,q})) = 0$.

(iv) $SO\left(L\left(K_{p,q}\right), x\right) = \frac{1}{2}pq(p+q-2)x^{\sqrt{2}(p+q-2)}$.

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(v) $RSO\left(L\left(K_{p,q}\right),x\right)=\frac{1}{2}pq\left(p+q-2\right)x^{\sqrt{2}(p+q-3)}.$

(vi) $ASO\left(L\left(K_{p,q}\right),x\right)=\frac{1}{2}pq\left(p+q-2\right)x^{0}.$

Proof: Let $K_{p,q}$ be a complete bipartite graph with $p+q$ vertices, pq edges and $1\leq p\leq q$. Then the line graph $L(K_{p,q})$ of $K_{p,q}$ is an $(p+q-2)$-regular graph with pq vertices and $\frac{1}{2}pq(p+q-2)$ edges. Therefore $\Delta\left(L\left(K_{p,q}\right)\right)=\delta\left(L\left(K_{p,q}\right)\right)=p+q-2$ and $d_{L(K_{p,q})}(u)=p+q-2$ for any vertex u of $L(K_{p,q})$.

By using definitions, we derive

(i) $SO(L(K_{p,q})) = \frac{1}{2}pq(p+q-2)\sqrt{(p+q-2)^2+(p+q-2)^2}$

$= \frac{1}{\sqrt{2}}pq(p+q-2)^2.$

(ii) $RSO(L(K_{p,q})) = \frac{1}{2}pq(p+q-2)\sqrt{(p+q-2-1)^2+(p+q-2-1)^2}$

$= \frac{1}{2}pq(p+q-2)(p+q-3).$

(iii) $ASO(L(K_{p,q})) = 0$, since $L(K_{p,q})$ is regular.

(iv) $SO(L(K_{p,q}), x) = \frac{1}{2}pq(p+q-2)x^{\sqrt{(p+q-2)^2+(p+q-2)^2}}$

$= \frac{1}{2}pq(p+q-2)x^{\sqrt{2}(p+q-2)}.$

(v) $RSO(L(K_{p,q}), x) = \frac{1}{2}pq(p+q-2)x^{\sqrt{(p+q-2-1)^2+(p+q-2-1)^2}}$

$= \frac{1}{2}pq(p+q-2)x^{\sqrt{2}(p+q-3)}.$

(vi) $ASO(L(K_{p,q}), x) = \frac{1}{2}pq(p+q-2)x^{0}.$

Using Theorem 2, we obtain the following results.

Corollary 2.1. Let $K_{p,p}$ be a complete bipartite graph. Then

(i) $SO(L(K_{p,p})) = 2\sqrt{2}p^2(p-1)^2.$

(ii) $RSO(L(K_{p,p})) = \sqrt{2}p^2(p-1)(2p-3).$

(iii) $ASO(L(K_{p,p})) = 0.$

(iv) $SO(L(K_{p,p}), x) = p^2(p-1)x^{2\sqrt{2}(p-1)}$

(v) $RSO(L(K_{p,p}), x) = p^2(p-1)x^{\sqrt{2}(2p-3)}$

(vi) $ASO(L(K_{p,p}), x) = p^2(p-1)x^{0}.$

Corollary 2.2. Let $K_{1,p}$ be a star. Then

(i) $SO(L(K_{1,p})) = \frac{1}{\sqrt{2}}p(p-1)^2.$

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(ii) $RSO(L(K_{1,p})) = \frac{1}{\sqrt{2}} p(p-1)(p-3).$

(iii) $ASO(L(K_{1,p})) = 0.$

(iv) $SO(L(K_{1,p}), x) = \frac{1}{2} p(p-1) x^{\sqrt{2}(p-1)}$

(v) $RSO(L(K_{1,p}), x) = \frac{1}{2} p(p-1) x^{\sqrt{2}(p-2)}$

(vi) $ASO(L(K_{1,p}), x) = \frac{1}{2} p(p-1) x^{0}.$

3. RESULTS FOR SUBDIVISION GRAPHS

The subdivision graph $S(G)$ of a graph G is the graph obtained from G by replacing each of its edges by a path of length two.

In the following theorem, we determine the Sombor indices and their exponentials of the subdivision graphs of r-regular graphs.

Theorem 3. Let G be an r-regular graph with $n \geq 3$ vertices. Then

(i) $SO(S(G)) = nr\sqrt{4+r^2}.$

(ii) $RSO(S(G)) = nr\sqrt{2-2r+r^2}.$

(iii) $ASO(S(G)) = \frac{r}{2+r}\sqrt{(4n-2nr)^2 + (nr^2-2nr)^2}.$

(iv) $SO(S(G), x) = nrx^{\sqrt{4+r^2}}.$

(v) $RSO(S(G), x) = nrx^{\sqrt{2-2r+r^2}}.$

(vi) $ASO(S(G), x) = nrx^{\frac{\sqrt{(4n-2nr)^2+(nr^2-2nr)^2}}{2n+nr}}.$

Proof: Let G be an r-regular graph with $n \geq 3$ vertices. Then the subdivision graph $S(G)$ of G has $n + \frac{nr}{2}$ vertices and nr edges. The edge partition of $S(G)$ is as follows.

$$E = \{uv \in E(S(G)) \mid d_{S(G)}(u) = 2, d_{S(G)}(v) = r\}, \qquad |E| = nr.$$

By using definitions, we obtain

(i) $SO(S(G)) = nr\sqrt{2^2 + r^2} = nr\sqrt{4+r^2}$

(ii) $RSO(S(G)) = nr\sqrt{(2-1)^2 (r-1)^2} = nr\sqrt{2-2r+r^2}$

(iii) $ASO(S(G)) = nr\sqrt{\left(2-\frac{4nr}{2n+nr}\right)^2 \left(r-\frac{4nr}{2n+nr}\right)^2} = \frac{r}{2+r}\sqrt{(4n-2nr)^2 + (nr^2-2nr)^2}$

(iv) $SO(S(G), x) = nrx^{\sqrt{2^2+r^2}} = nrx^{\sqrt{4+r^2}}$

(v) $RSO(S(G), x) = nrx^{\sqrt{(2-1)^2+(r-1)^2}} = nrx^{\sqrt{2-2r+r^2}}$

(vi) $ASO(S(G), x) = nrx^{\sqrt{\left(2-\frac{4nr}{2n+nr}\right)^2+\left(r-\frac{4nr}{2n+nr}\right)^2}} = nrx^{\sqrt{\frac{(4n-2nr)^2+(nr^2-2nr)^2}{2n+nr}}}$

From Theorem 3, we obtain the following results.

Corollary 3.1. Let C_n be a cycle with $n\geq 3$ vertices. Then

(i) $SO(S(C_n)) = 4\sqrt{2}n$

(ii) $RSO(S(C_n)) = 2\sqrt{2}n$

(iii) $ASO(S(C_n)) = 0$.

(iv) $SO(S(C_n), x) = 2nx^{2\sqrt{2}}$

(v) $RSO(S(C_n), x) = 2nx^{\sqrt{2}}$

(vi) $ASO(S(C_n), x) = 2nx^{0}$

Corollary 3.2. Let K_n be a complete graph.

(i) $SO(S(K_n)) = n(n-1)\sqrt{5-2n+n^2}$.

(ii) $RSO(S(K_n)) = n(n-1)\sqrt{5-4n+n^2}$.

(iii) $ASO(S(K_n)) = \dfrac{n-1}{n+1}\sqrt{\left(6n-2n^2\right)^2+\left(n^3-4n^2+3n\right)^2}$.

(iv) $SO(S(K_n), x) = n(n-1)x^{\sqrt{5-2n+n^2}}$.

(v) $RSO(S(K_n), x) = n(n-1)x^{\sqrt{5-4n+n^2}}$

(vi) $ASO(S(K_n), x) = n(n-1)x^{\frac{\sqrt{\left(6n-2n^2\right)^2+\left(n^3-4n^2+3n\right)^2}}{n(n+1)}}$.

In the following theorem, we compute the Sombor indices and their exponentials of the subdivision graphs of complete bipartite graphs.

Theorem 4. Let $K_{p,q}$ be a complete bipartite graph with $p+q$ vertices, pq edges and $1 \leq p \leq q$. Then

(i) $SO\left(S\left(K_{p,q}\right)\right) = pq\sqrt{p^2+4} + pq\sqrt{q^2+4}$.

(ii) $RSO\left(S\left(K_{p,q}\right)\right) = pq\sqrt{p^2+q-2} + pq\sqrt{q^2-2q+4}$.

(iii) $ASO\left(S\left(K_{p,q}\right)\right) = \dfrac{pq}{p+q+pq}\left[\left(p^2+p^2q-3pq\right)^2+\left(2p+2q-2pq\right)^2\right]^{\frac{1}{2}}$

$+\dfrac{pq}{p+q+pq}\left[\left(q^2+pq^2-3pq\right)^2+\left(2p+2q-2pq\right)^2\right]^{\frac{1}{2}}$

(iv) $SO\left(S\left(K_{p,q}\right),x\right) = pqx^{\sqrt{p^2+4}} + pqx^{\sqrt{q^2+4}}$.

(v) $RSO\left(S\left(K_{p,q}\right),x\right) = pqx^{\sqrt{p^2-2p+2}} + pqx^{\sqrt{q^2-2q+2}}$.

(vi) $ASO\left(S\left(K_{p,q}\right),x\right) = pqx^{\frac{\left[\left(p^2+p^2q-3pq\right)^2+\left(2p+2q-2pq\right)^2\right]^{\frac{1}{2}}}{p+q+pq}} + pqx^{\frac{\left[\left(q^2+pq^2-3pq\right)^2+\left(2p+2q-2pq\right)^2\right]^{\frac{1}{2}}}{p+q+pq}}$.

Proof: Let $K_{p,q}$ be a complete bipartite graph with $p+q$ vertices, pq edges and $1\leq p\leq q$. The vertex set of $K_{p,q}$ can be partitioned into two independent sets V_1 and V_2 such that $u\in V_1$ and $u\in V_2$ for every edge uv of $K_{p,q}$. Let $K=K_{p,q}$. We have $d_K(u)=q$ and $d_K(v)=p$. Then subdivision graph $S(K_{p,q})$ has $p+q+pq$ vertices and 2pq edges. The edge partition of $S(K_{p,q})$ is as follows:

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$E_1 = \{uv \in E(K) \mid d_K(u) = p, d_K(v) = 2\}$ $|E_1| = pq$.

$E_2 = \{uv \in E(K) \mid d_K(u) = q, d_K(v) = 2\}$ $|E_2| = pq$.

By using definitions, we derive

(i) $SO(S(K_{p,q})) = pq\sqrt{p^2+2^2} + pq\sqrt{q^2+2^2}$
$= pq\sqrt{p^2+4} + pq\sqrt{q^2+4}$

(ii) $RSO(S(K_{p,q})) = pq\sqrt{(p-1)^2+(2-1)^2} + pq\sqrt{(q-1)^2+(2-1)^2}$
$= pq\sqrt{p^2-2p+2} + pq\sqrt{q^2-2q+2}$

(iii) $ASO(S(K_{p,q})) = pq\left[\left(p-\frac{4pq}{p+q+pq}\right)^2+\left(2-\frac{4pq}{p+q+pq}\right)^2\right]^{\frac{1}{2}}$

$+pq\left[\left(q-\frac{4pq}{p+q+pq}\right)^2+\left(2-\frac{4pq}{p+q+pq}\right)^2\right]^{\frac{1}{2}}$

$=\frac{pq\left[\left(p^2+p^2q-3pq\right)^2+\left(2q+2q-2pq\right)^2\right]^{\frac{1}{2}}}{p+q+pq}$

$+\frac{pq\left[\left(q^2+pq^2+pq\right)^2+\left(2q+2q-2pq\right)^2\right]^{\frac{1}{2}}}{p+q+pq}$

(iv) $SO(S(K_{p,q}), x) = pqx^{\sqrt{p^2+4}} + pqx^{\sqrt{q^2+4}}$

(v) $RSO(S(K_{p,q}), x) = pqx^{\sqrt{p^2-2p+2}} + pqx^{\sqrt{q^2-2q+2}}$

(vi) $ASO(S(K_{p,q}), x) = pqx^{\frac{\left[\left(p^2+p^2q-3pq\right)^2+\left(2q+2q-2pq\right)^2\right]^{\frac{1}{2}}}{p+q+pq}} + pqx^{\frac{\left[\left(q^2+pq^2-3pq\right)^2+\left(2p+2q-2pq\right)^2\right]^{\frac{1}{2}}}{p+q+pq}}$

From Theorem 4, we get the following results.

Corollary 4.1. Let $K_{p,p}$ be a complete bipartite graph. Then

(i) $SO(S(K_{p,p})) = 2p^2\sqrt{p^2+4}$.

(ii) $RSO(S(K_{p,p})) = 2p^2\sqrt{p^2-2p+2}$.

(iii) $ASO(S(K_{p,p})) = \frac{4p^2}{2+p}\left(p^4-p^3+2p^2-4p+4\right)^{\frac{1}{2}}$.

(iv) $SO(S(K_{p,p}), x) = 2p^2x^{\sqrt{p^2+4}}$.

(v) $RSO(S(K_{p,p}), x) = 2p^2x^{\sqrt{p^2-2p+2}}$.

(vi) $ASO(S(K_{p,p}), x) = \frac{2p}{2+p}x^{2p\left(p^4-p^3+2p^2-4p+4\right)^{\frac{1}{2}}}$.

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Corollary 4.2. Let $K_{1,p}$ be a star. Then

(i) $SO(S(K_{1,p})) = p\left(\sqrt{5}+\sqrt{p^2+1}\right).$

(ii) $RSO(S(K_{1,p})) = p\left(1+\sqrt{p^2-2p+2}\right).$

(iii) $ASO(S(K_{1,p})) = \frac{p}{1+2p}\left[\left(4p^2-4p+5\right)^{\frac{1}{2}}+\left(4p^4-12p^3+9p^2+4\right)^{\frac{1}{2}}\right].$

(iv) $SO(S(K_{1,p}), x) = px^{\sqrt{5}} + px^{\sqrt{p^2+4}}$

(v) $RSO(S(K_{1,p}), x) = px^1 + px^{\sqrt{p^2-2p+2}}.$

(vi) $ASO(S(K_{1,p}), x) = \frac{p}{1+2p}x^{\left(4p^2-4p+5\right)^{\frac{1}{2}}} + \frac{p}{1+2p}x^{\left(4p^4-12p^3+9p^2+4\right)^{\frac{1}{2}}}.$

CONCLUSION

Gutman considered a class of novel graph invariants of which the Sombor index was introduced. In this paper, we have determined the certain Sombor indices and their corresponding exponentials of regular and complete bipartite graphs using graph operators such as line graph and subdivision graph.

REFERENCES

[1] V.R.Kulli, *College Graph Theory*, Vishwa International Publications, Gulbarga, India (2012).
[2] I.Gutman, Geometric approach to degree based topological indices : Sombor indices *MATCH Common, Math. Comput. Chem.* 86(2021) 11-16.
[3] V.R.Kulli, The (a, b)-KA indices of polycyclic aromatic hydrocarbons, and benzenoid systems, *International Journal of Mathematics Trends and Technology*, 65(11) (2019) 115-120.
[4] V.R.Kulli, Graph indices, in *Hand Book of Research on Advanced Applications of Application Graph Theory in Modern Society*, M. Pal. S. Samanta and A. Pal, (eds.) IGI Global, USA (2020) 66-91.
[5] R.Todeschini and V. Consonni, *Molecular Descriptors for Chemoinformatics*, Wiley-VCH, Weinheim, (2009).
[6] S.Ediz, On the reduced first Zagreb index of graphs, Pacific J. Appl. Math. 8(2) (2016) 99-102.
[7] B. Furtula, I. Gutman and S. Ediz, On difference of Zagreb indices, *Discrete Appl. Math*. 178, (2014) 83-88.